Fermionic Zero Modes and Spontaneous Symmetry Breaking on the Light Front

Ľubomír Martinovič
Institute of Physics, Slovak Academy of Sciences
Dúbravská cesta 9, 842 28 Bratislava, Slovakia *
and
International Institute of Theoretical and Applied Physics
Iowa State University, Ames, Iowa 50011, USA

and
James P. Vary
Department of Physics and Astronomy
and
International Institute of Theoretical and Applied Physics,
Iowa State University, Ames, Iowa 50011, USA

February 1, 2008

Abstract

Spontaneous symmetry breaking is studied within a simple version of the light-front $O(2)$ sigma model with fermions. Its vacuum structure is derived by an implementation of global symmetries in terms of unitary operators in a finite volume with periodic fermi field. Due to the dynamical fermion zero mode, the vector and axial $U(1)$ charges do not annihilate the light-front vacuum. The latter is transformed into a continuous set of degenerate vacuum states, leading to the spontaneous breakdown of the axial symmetry. The existence of associated massless Nambu-Goldstone boson is demonstrated.

The phenomenon of spontaneous symmetry breaking represents a challenge in the light-front (LF) formulation of quantum field theory. In contrast to the usual quantization on space-like surfaces, the vacuum of the theory quantized on the surface of the constant LF time x^+ (i.e. on the light front) can be defined kinematically as a state with minimum (zero) longitudinal LF momentum p^+, since the operator P^+ has a positive spectrum [1]. Thus, neglecting modes of quantum fields with $p^+ = 0$ (zero modes – ZM), the vacuum state of even the interacting theory does not contain dynamical quanta. This "triviality" of the ground state is very advantageous for the Fock-state description of the bound states [2], but it seems to forbid such important non-perturbative aspects like vacuum degeneracy and formation of condensates. Since there is no a priori

*permanent address

reason to expect any inconsistency [1,3] in Dirac's front form of relativistic dynamics [4,5], it should be a sensible strategy to look for a genuine LF description of symmetry breaking [6] and related aspects of the vacuum structure [7], which would complement the usual space-like formulation based on a very complex dynamical vacuum state. Note in this context that, on the LF, in contrast to the space-like quantization [8,9], even those charges which correspond to non-conserved currents do annihilate the vacuum in the continuum theory [10,11]. Thus, one may expect similar "surprises" in other aspects of the LF field theory.

A convenient regularized framework for studying these and related problems of non-perturbative nature is quantization in a finite volume with fields obeying periodic boundary conditions. It allows one to separate infrared aspects (ZM operators relevant for vacuum properties) from the remainder of the dynamics [12]. Note that to have a well-defined theory, one has to specify boundary conditions also in the continuum formulation [13,14].

For self-interacting LF scalar theories a bosonic ZM is not a dynamical degree of freedom [12] but a constrained variable. Thus the vacuum remains indeed "empty" and one expects that physics of spontaneous symmetry breaking (SSB) is contained in solutions of a complicated operator ZM constraint [15,16]. If a continuous symmetry is spontaneously broken, a massless Nambu-Goldstone (NG) boson should be present in the spectrum of states. However, as has been emphasized by Yamawaki and collaborators [6], the Goldstone theorem cannot exist on the light front as long as all charges annihilate the LF vacuum. Instead, a singular behaviour of the NG field and the charge non-conservation in the massless limit of a regularized theory has been identified as the manifestation of the NG phase in the LF scalar theories.

The situation is different however for LF fermions in (3+1) dimensions. A massless fermion field, when quantized in a finite volume with periodic boundary conditions, contains a global ZM which is a dynamical variable. Recently, it has been demonstrated within the massive LF Schwinger model with antiperiodic fermion field that the residual symmetry under large gauge transformations, when realized quantum mechanically, gives rise to a non-trivial vacuum structure in terms of gauge-field zero mode as well as of fermion excitations [7]. It is a purpose of the present work to demonstrate that dynamical fermion ZM provides a similar mechanism for a simple non-gauge field theory with fermions. Charges, which are the generators of global symmetries of the given system, contain a ZM part and consequently transform the trivial vacuum into a continuous set of degenerate vacuum states. This leads to a SSB in the usual sense [17–23] with non-zero vacuum expectation values of certain operators and a massless NG state in the spectrum of states. Much of what we demonstrate is of a rather general nature.

In the LF field theory, the dynamical symmetry breaking [17,18] has been studied so far within the usual mean-field approximation [24,25] and also by means of Schwinger-Dyson equations [26].

To simplify our discussion of SSB in the LF field theory as much as possible, we will consider a version of the $O(2)$-symmetric sigma model with fermions

[6,28] specified by the Lagrangian density

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(\frac{i}{2}\gamma^{\mu} \overleftrightarrow{\partial_{\mu}} - m)\psi + \frac{1}{2}(\partial_{\mu}\sigma\partial^{\mu}\sigma + \partial_{\mu}\pi\partial^{\mu}\pi) \\ &- \frac{1}{2}\mu^2(\sigma^2+\pi^2) - g\bar{\psi}(\sigma + i\gamma^5\pi)\psi, \end{aligned} \tag{1}$$

where the quartic self-interaction term for the scalar fields σ and π has been omitted, because it is not relevant for our purpose. The Lagrangian (1) is invariant under the global $U(1)$ transformation $\psi \rightarrow \exp(-i\alpha)\psi$ and for $m = 0$ also under the axial transformation

$$\psi \rightarrow \exp(-i\beta\gamma^5)\psi, \quad \psi^{\dagger} \rightarrow \psi^{\dagger}\exp(i\beta\gamma^5), \tag{2}$$

$$\sigma \rightarrow \sigma\cos 2\beta - \pi \sin 2\beta, \quad \pi \rightarrow \sigma \sin 2\beta + \pi\cos 2\beta. \tag{3}$$

Rewriting the above Lagrangian in terms of the LF variables, one finds for the LF Hamiltonian

$$\begin{aligned} P^{-} &= \int_V d^3\underline{x}\,\Big[(\partial_k\sigma)^2 + (\partial_k\pi)^2 + \mu^2(\sigma^2+\pi^2) + \psi_{+}^{\dagger}(m\gamma^0 \\ &- i\alpha^k\partial_k)\psi_{-} + g\psi_{+}^{\dagger}\gamma^0(\sigma + i\gamma^5\pi)\psi_{-} + h.c.\Big], \end{aligned} \tag{4}$$

where $d^3\underline{x} \equiv \frac{1}{2}dx^{-}d^2x^{\perp}$. Our convention for LF coordinates is $x^{\pm} = x^0 \pm x^3$, $p^{\mu}x_{\mu} = \frac{1}{2}p^{-}x^{+} + \underline{p}\underline{x}$, $\underline{p}\underline{x} = \frac{1}{2}p^{+}x^{-} - x^{\perp}p^{\perp}, x^{\perp}p^{\perp} \equiv x^k p^k, k = 1,2$ and x^{+}, p^{-} are the LF time and energy. Correspondingly, we define the Dirac matrices as $\gamma^{\pm} = \gamma^0 \pm \gamma^3$, $\alpha^k = \gamma^0\gamma^k$, the LF projection operators as $\Lambda_{\pm} = \frac{1}{2}\gamma^0\gamma^{\pm}$ and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. $\Lambda_{\pm}$ separate the fermi field into the independent component $\psi_{+} = \Lambda_{+}\psi$ and the dependent one $\psi_{-} = \Lambda_{-}\psi$.

The infrared-regularized formulation is achieved by enclosing the system into a three-dimensional box $-L \le x^{-} \le L, -L_{\perp} \le x^k \le L_{\perp}$ with volume $V = 2L(2L_{\perp})^2$ and by imposing periodic boundary conditions for all fields in $x^{-}, x^{\perp}$. This leads to a decomposition of the fields into the zero-mode (subscript 0) and normal-mode (NM, subscript n) parts. One finds that $\psi_{-}, \psi_{-}^{\dagger}, \sigma_0, \pi_0$ are non-dynamical fields with vanishing conjugate momenta, while $\psi_{+}, \psi_{+}^{\dagger}, \sigma_n, \pi_n$ are dynamical. For a consistent quantization, one should apply the Dirac-Bergmann or a similar method suitable for systems with constraints. We will postpone that for a more detailed work [27], assuming here the standard (anti)commutators at $x^{+} = 0$:

$$\{\psi_{+i}(\underline{x}), \psi_{+j}^{\dagger}(\underline{y})\} = \frac{1}{2}\delta_{ij}\delta^3(\underline{x} - \underline{y}), \; i,j = 1,4, \tag{5}$$

$$[\phi_n(\underline{x}), \partial_{-}\phi_n(\underline{y})] = \frac{i}{2}\delta_n^3(\underline{x} - \underline{y}). \tag{6}$$

We are working in chiral representation with diagonal γ^5 and $\phi = \sigma$ or π. The anticommutator (5) can be derived by a direct calculation based on the field expansion

$$\psi_{+}(\underline{x}) = \sum_{\substack{p^{+},p^{\perp} \\ s=\pm\frac{1}{2}}} \frac{u(s)}{\sqrt{V}}(b(\underline{p},s)e^{-i\underline{p}\underline{x}} + d^{\dagger}(\underline{p},-s)e^{i\underline{p}\underline{x}}), \tag{7}$$

$$\{b(\underline{p},s), b^{\dagger}(\underline{p}',s')\} = \{d(\underline{p},s), d^{\dagger}(\underline{p}',s')\} = \delta_{s,s'}\delta_{\underline{p},\underline{p}'}. \tag{8}$$

Here and in the Fourier representation of the periodic delta function $\delta^3(\underline{x}-\underline{y}) = \delta_0 + \delta_n^3(\underline{x}-\underline{y}), \delta_0 = \frac{2}{V}$, the summations run over discrete momenta $p^+ = 2\pi L^{-1}n,\ n = 0, 1\ldots, N \to \infty,\ p^k = \pi L_\perp^{-1} n^k,\ n^k = 0, \pm 1, \ldots, \pm N_\perp \to \infty$. The spinors are $u^\dagger(s=\frac{1}{2}) = (1\ 0\ 0\ 0), u^\dagger(s=-\frac{1}{2}) = (0\ 0\ 0\ 1)$ with s being the LF helicity.

The non-dynamical fields satisfy the constraints

$$2i\partial_-\psi_- = \Big[m\gamma^0 - i\alpha^k\partial_k + g\gamma^0\left(\sigma + \gamma^5\pi\right)\Big]\psi_+, \tag{9}$$

$$(\partial_k\partial_k - \mu^2)\sigma_0 = g\int_{-L}^{+L}\frac{dx^-}{2L}(\psi_+^\dagger\gamma^0\psi_- + h.c.), \tag{10}$$

$$(\partial_k\partial_k - \mu^2)\pi_0 = g\int_{-L}^{+L}\frac{dx^-}{2L}(i\psi_+^\dagger\gamma^0\gamma^5\psi_- + h.c.). \tag{11}$$

The fermion constraint (9) requires the dynamical fermion ZM ψ_{+0} to vanish in the free massive theory. For the free massless fermi field, only the $\underline{x}$-independent *global* ZM is compatible with the constraint. Decomposing σ_0, π_0 into the *proper* zero modes [29] $\sigma_0(x^\perp), \pi_0(x^\perp)$ (which will not be needed here) and the global ZM $\hat{\sigma}_0, \hat{\pi}_0$, the above constraints can be projected into three global-ZM sector relations. In Eqs.(10) and (11), we have assumed the existence of the ψ_{-0} zero mode, so the integrands are given by the diagonal combinations $\psi_{+0}\gamma^0\psi_{-0} + \psi_{+n}\gamma^0\psi_{-n}$, etc. Actually, by combining the global ZM constraints (with $m = 0$) into one

$$\begin{aligned}&\Big(\psi_{+0}^\dagger\gamma^0\psi_{-0} + h.c.\Big)\psi_{+0} - \Big(\psi_{+0}^\dagger\gamma^0\gamma^5\psi_{-0} - h.c.\Big)\gamma^5\psi_{+0}\\ &+\int_V\frac{d^3\underline{x}}{V}\Big[(\psi_{+n}^\dagger\gamma^0\psi_{-n} + h.c.) - (\psi_{+n}^\dagger\gamma^0\gamma^5\psi_{-n}\\ &-h.c.)\gamma^5\Big]\psi_{+0} = \frac{\mu^2}{g}\int_V\frac{d^3\underline{x}}{V}\Big(\sigma_n + i\pi_n\gamma^5\Big)\psi_{+n},\end{aligned} \tag{12}$$

we see that non-zero ψ_{-0} is *required* for consistency: setting $\psi_{-0} = 0$ in Eq.(12) yields an operator relation among independent fields which cannot be satisfied.

ψ_{-n} can be determined from the constraint (9):

$$\begin{aligned}\psi_{-n}(\underline{x}) =& \frac{1}{4i}\int_{-L}^{+L}\frac{dy^-}{2}\epsilon_n(x^- - y^-)\Big\{(m\gamma^0 - i\alpha^k\partial_k)\psi_{+n}(\underline{y})\\ &+g\gamma^0\Big[\Big(\sigma_n(\underline{y}) + i\pi_n(\underline{y})\gamma^5\Big) + \Big(\sigma_0 + i\pi_0\gamma^5\Big)\Big]\Big\}\psi_+(\underline{y}),\end{aligned} \tag{13}$$

where $\epsilon_n(x^- - y^-)$ is the normal-mode part of the periodic sign function and $\underline{y} \equiv (y^-, x^\perp)$. Due to the presence of σ_0, π_0, which in turn are given by their own constraints (10),(11) depending on ψ_{-n}, it is difficult to solve (13) in a closed form. Iterative solutions are possible and the lowest order one is obtained by setting $\sigma_0 = \pi_0 = 0$.

While the free massive fermion Hamiltonian is, unlike the space-like quantization, symmetric under the axial vector transformations (15) below [30], the

mass term in the ψ_--constraint generates interaction terms which are proportional to mg and which, due to an extra γ^0, violate the axial symmetry explicitly. This is the reason why we shall set $m = 0$ henceforth. Note however that the scalar fields have to be massive [6] to avoid infrared problems.

Inserting the ψ_{-n}-constraint into (4), we find

$$
\begin{aligned}
P^-_{int} &= \int_V d^3\underline{x}\, \Big[\mu^2(\sigma_0^2 + \pi_0^2) + (\partial_k\sigma_0)^2 + (\partial_k\pi_0)^2\Big] \\
&+ig \int_V d^3\underline{x}\ \psi^\dagger_+(\underline{x})\Sigma^\dagger(\underline{x}) \int\limits_{-L}^{+L} \frac{dy^-}{2}\frac{1}{2}\epsilon_n(x^- - y^-) \times \\
&\Big[i\gamma^k\partial_k\psi_{+n}(y^-,x^\perp) + h.c. - g\Sigma(y^-,x^\perp)\psi_+(y^-,x^\perp)\Big],
\end{aligned}
\tag{14}
$$

where $\Sigma(\underline{x}) \equiv \sigma(\underline{x}) + i\pi(\underline{x})\gamma^5$. It is not a closed expression due to the presence of $\hat\sigma_0, \hat\pi_0, \sigma_0(x^\perp), \pi_0(x^\perp)$. However, this is not an obstacle for determining the symmetry properties of the Hamiltonian, which are of primary importance in the present approach. First, we observe that the LF analogue of the axial vector transformation (2) is

$$
\psi_+(\underline{x}) \to \exp{(-i\beta\gamma^5)}\psi_+(\underline{x}), \tag{15}
$$

while the NM fields σ_n, π_n transform according to (3). As for the constrained variables, we shall demand that ψ_{-n} has a well defined transformation law, which is unambiguously fixed by the terms with α^k, σ_n and π_n in the solution (13). It follows that $\sigma_0 + i\pi_0\gamma^5$ will transform exactly as $\sigma_n + i\pi_n\gamma^5$ and that the whole ψ_{-n} will transform for $m = 0$ in the same way as ψ_+. As a result, we find that P^-_{int} is invariant under $U_A(1)$ transformations in addition to $U(1)$.

These symmetries give rise to the conserved (normal-ordered) vector current $j^\mu =: \psi^\dagger\gamma^0\gamma^\mu\psi :,\ \partial_\mu j^\mu = 0$ and the conserved axial-vector current $j^\mu_5 = \psi^\dagger\gamma^0\gamma^\mu\gamma^5\psi + 2(\sigma\partial^\mu\pi - \pi\partial^\mu\sigma)$ $(\mu = +,-,k)$:

$$
\partial_\mu j^\mu_5 = 2m\Big(i\psi^\dagger_+\gamma^0\gamma^5\psi_- + h.c.\Big) = 0 \ \text{ for } \ m = 0. \tag{16}
$$

They are implemented by the unitary operators $U(\alpha) = \exp(-i\alpha Q)$, $V(\beta) = \exp(-i\beta Q^5)$:

$$
\begin{aligned}
\psi_+(\underline{x}) &\to e^{-i\alpha}\psi_+(\underline{x}) = U(\alpha)\psi_+(\underline{x})U^\dagger(\alpha), \\
\psi_+(\underline{x}) &\to e^{-i\beta\gamma^5}\psi_+(\underline{x}) = V(\beta)\psi_+(\underline{x})V^\dagger(\beta).
\end{aligned}
\tag{17}
$$

While the NM parts of the charge operators Q and Q^5

$$
Q = \int_V d^3\underline{x} j^+(\underline{x}) = 2\int_V d^3\underline{x}\psi^\dagger_+\psi_+, \tag{18}
$$

$$
Q^5 = 2\int_V d^3\underline{x}\Big[\psi^\dagger_+\gamma^5\psi_+ + 2\Big(\sigma_n\partial_-\pi_n - \pi_n\partial_-\sigma_n\Big)\Big] \tag{19}
$$

are diagonal in creation and annihilation operators, the ZM parts, which do not vanish in the free nor the interacting theory, contain also off-diagonal terms

$$
\begin{aligned}
Q_0 \ &= \ \sum_s [b^\dagger_0(s)b_0(s) - d^\dagger_0(s)d_0(s) \\
&+ \ b^\dagger_0(s)d^\dagger_0(-s) + d_0(s)b_0(-s)],
\end{aligned}
\tag{20}
$$

$$\begin{aligned} Q_0^5 &= \sum_s 2s[b_0^\dagger(s)b_0(s) + d_0^\dagger(s)d_0(s) \\ &+ b_0^\dagger(s)d_0^\dagger(-s) - d_0(s)b_0(-s)]. \end{aligned} \tag{21}$$

The commuting ZM charges Q_0, Q_0^5 *do not* annihilate the LF vacuum $|0\rangle$ defined by $b(\underline{p}, s)|0\rangle = d(\underline{p}, s)|0\rangle = 0$. However, their vacuum expectation values are zero as they have to be. In this way, the vacuum of the model transforms under $U(\alpha), V(\beta)$ as $|0\rangle \rightarrow |\alpha\rangle = \exp(-i\alpha Q_0)|0\rangle,\ |0\rangle \rightarrow |\beta\rangle = \exp(-i\beta Q_0^5)|0\rangle$, where

$$|\alpha\rangle = \exp\left(-i\alpha\sum_s\left[b_0^\dagger(s)d_0^\dagger(-s) + h.c.\right]\right)|0\rangle, \tag{22}$$

$$|\beta\rangle = \exp\left(-i\beta\sum_s 2s\left[b_0^\dagger(s)d_0^\dagger(-s) + h.c.\right]\right)|0\rangle. \tag{23}$$

The vacua contain ZM fermion-antifermion pairs with opposite helicities. Due to Fermi-Dirac statistics, the number of such "Cooper pairs" cannot exceed two.

Thus, the global symmetry of the Hamiltonian (14) leads to an infinite set of translationally invariant states $|\alpha, \beta\rangle = U(\alpha)V(\beta)|0\rangle$ ($P^+|\alpha, \beta\rangle = P^\perp|\alpha, \beta\rangle = 0$), labeled by two real parameters. Since $U(\alpha), V(\beta)$ commute with P^-, the vacua are degenerate in the LF energy. The Fock space can be built from any of them since they are unitarily equivalent.

We are in a position now to demonstrate the existence of the Goldstone theorem in the considered model. We have all the ingredients for the usual proof of the theorem [19–23]: the existence of the conserved current j_5^μ, the operators A, namely $\bar\psi\psi = \psi_+^\dagger\gamma^0\psi_- + \psi_-^\dagger\gamma^0\psi_+$ and $\bar\psi\gamma^5\psi = \psi_+^\dagger\gamma^0\gamma^5\psi_- + \psi_-^\dagger\gamma^0\gamma^5\psi_+$, which are non-invariant under the axial transformation

$$A \rightarrow V(\beta)AV^\dagger(\beta) \neq A \Rightarrow \delta A = -i\beta[Q^5, A] \neq 0, \tag{24}$$

and the property $Q^5|\alpha, \beta\rangle = Q_0^5|\alpha, \beta\rangle \neq 0$. Of course, the above fermi bilinears are symmetric under $U(1)$, so the commutator $[Q, A]$ vanishes and there is no symmetry breaking associated with this symmetry.

In a little more detail, from the axial current conservation and the periodicity in $x^-, x^\perp$ we get

$$\partial_+\langle vac|[Q^5(x^+), A]|vac\rangle = 0, \quad |vac\rangle \equiv |\alpha, \beta\rangle \tag{25}$$

in addition to

$$\langle vac|[Q^5(x^+), A]|vac\rangle \neq 0. \tag{26}$$

These expressions imply that the the vacuum expectation value of the above commutator is a time-independent quantity. Note that the relation (26) is only possible due to the fact that Q^5 does not annihilate the vacuum and this crucially depends on the existence of the ZM part of Q^5. Inserting now a complete set of four-momentum eigenstates into the Eqs.(25) and (26) and using the translational invariance

$$e^{-iP_\mu x^\mu}|vac\rangle = |vac\rangle,\ j_5^+(x) = e^{-iP_\mu x^\mu}j_5^+(0)e^{iP_\mu x^\mu} \tag{27}$$

we arrive in the usual way [19,20,22,23] to the conclusion that there must exist a state $|n\rangle = |G\rangle$ such, that

$$\langle vac|A|G\rangle\langle G|j_5^+(0)|vac\rangle \neq 0 \tag{28}$$

with $P^-_{\rm G} = 0$ for $P^+_{\rm G} = P^\perp_{\rm G} = 0$. Thus, $M^2_{\rm G} = P^+_{\rm G}P^-_{\rm G} - (P^\perp_{\rm G})^2 = 0$. From the infinitesimal rotation of the Fock vacuum we have explicitly

$$Q^5_0|0\rangle = \sum_s 2sb^\dagger_0(s)d^\dagger_0(-s)|0\rangle \equiv |G\rangle. \tag{29}$$

Using the transformation law of the $\psi_\pm$ fields and the anticommutator (5), one can show that the relation (26) implies non-zero vacuum expectation values of the operators A [27]. They will depend on the coupling constant through ψ_{-n}. To obtain quantitative results, one has to solve approximately the constraint (9) [28].

To summarize, we have demonstrated that spontaneous symmetry breaking can occur in the *finite-volume* formulation of the fermionic LF field theory. While in contrast with the usual expectation within the space-like field theory (see [31], e.g.), this is related to the explicit presence of a dynamical fermion zero mode in the finite-volume LF quantization. One of the advantages of this infrared-regularized formulation is that one does not need to introduce test functions and complicated definitions of operators to obtain a mathematically rigorous framework [20]. For example, contrary to the standard infinite-volume formulation, the norm of the state $Q^5|vac\rangle = Q^5_0|vac\rangle$ is finite and *volume-independent*. However, the issue of continuum limit and volume independence of the physical picture obtained in a finite volume requires a further study.

In the usual treatment of fermionic theories [17,31,32], the considered vacua, related by a canonical transformation, are the free-field vacua corresponding to fermion fields with different masses. In the LF picture, such vacua are unitarily equivalent [1]. Our approach relates the vacuum degeneracy to the unitary operators implementing the symmetries, making use of the "triviality" of the LF vacuum in the sector of normal Fourier modes.

Nevertheless, there are still a few aspects of the present approach that have to be understood better. First, one has to perform a full constrained quantization of the model to derive the (anti)commutation relations for all relevant (ZM) degrees of freedom. Also, the connection of our picture with the standard one, based on the mean-field approximation and the new vacuum with lower energy above the critical coupling, has to be clarified.

This work has been supported by the grants VEGA 2/5085/98, NSF No. INT-9515511 and by the U.S. Department of Energy, Grant No. DE-FG02-87ER40371.

References

[1] H. Leutwyler, J.R. Klauder and L. Streit, Nuovo Cim.**66A**, 536 (1970).

[2] H.C. Pauli and S.J. Brodsky, Phys. Rev. D **32**, 1993 (1985).

[3] R. A. Neville and F. Rohrlich, Nuovo Cim.**1A**,625 (1971).

[4] P.A.M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[5] P. A. M. Dirac, in *Mathematical Foundations of Quantum Theory*, edited by A. Marlow (Academic Press, 1978).

[6] Y. Kim, S. Tsujimaru and K. Yamawaki, Phys. Rev. Lett. **74**, 4771 (1995); S. Tsujimaru and K. Yamawaki, Phys. Rev. D **57**, 4942 (1998).

[7] Ľ. Martinovič, hep-th/9811182.

[8] S. Coleman, J. Math. Phys. **7**, 787 (1966).

[9] E. Fabri and L. Picasso, Phys. Rev. Lett. **16**, 408 (1966).

[10] J. Jersák and J. Stern, Nuovo Cim. **59**, 315 (1969).

[11] H. Leutwyler, Springer Tracts in Mod. Phys. **50**, 29 (1969).

[12] T. Maskawa and K. Yamawaki, Prog. Theor. Phys. **56**, 270 (1976).

[13] P. Steinhardt, Ann. Phys. **128**, 425 (1980).

[14] Th. Heinzl, hep-th/9812190.

[15] C. M. Bender, S. S. Pinsky and B. van de Sande, Phys. Rev. D **48**, 816 (1993).

[16] T. Heinzl, C. Stern, E. Werner and B. Zellerman, Z. Phys. C 72, 353 (1996).

[17] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[18] J. Goldstone, Nuovo Cim. **XIX**, 154 (1961).

[19] J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. **127** 965 (1962).

[20] J. Swieca, in *Cargèse Lectures in Physics*, edited by D. Kastler (Gordon and Breach, 1969).

[21] G.S. Guralnik, C. R. Hagen and T. W. Kibble, in *Advances in Particle Physics*, edited by R. L. Cool and R. E. Marshak (Interscience, New York, 1968).

[22] C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, 1980).

[23] F. Strocchi, *Elements of Quantum Mechanics of Infinite Systems* (World Scientific, Singapore, 1985).

[24] Th. Heinzl, Phys. Lett. B**388**, 129 (1996).

[25] K. Itakura, Prog. Theor. Phys. **98**, 527 (1997).

[26] M. Burkardt and H. El-Khozondar, Phys. Rev. D **55**, 6514 (1997).

[27] Ľ. Martinovič and J. P. Vary, to be published.

[28] K. Itakura and S. Maedan, Phys. Rev. D **61**, 045009 (2000).

[29] A. Kalloniatis and H.C. Pauli, Z. Phys. C 63, 161 (1994).

[30] D. Mustaki, hep-ph/9404206.

[31] V. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories* (World Scientific, 1993).

[32] T. Hatsuda and T. Kunihiro, Phys. Rep. **274**, 221 (1994).